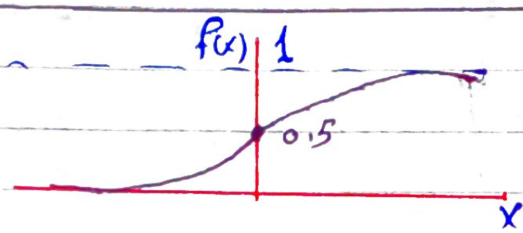
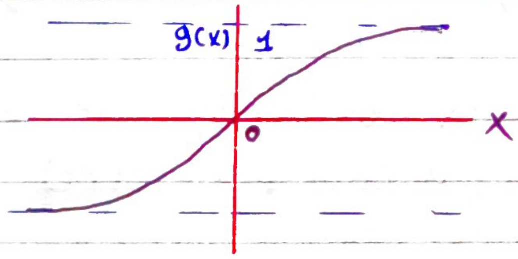
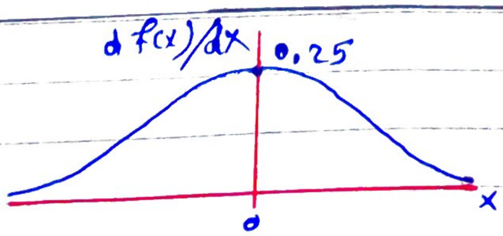
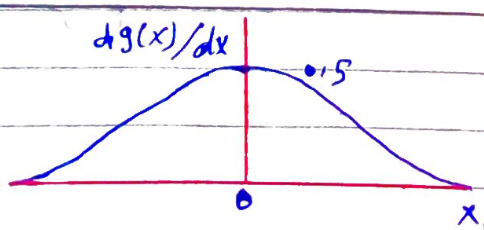
19/10/2016 الأربعاء د.محمود فهمي محاضرة $\boxed{4}$

A comparison between binary and bipolar sigmoidal functions.

	Binary Sigmoid	Bipolar Sigmoid
Definition	$f(x) = \dfrac{1}{1+e^{-x}}$	$g(x) = \dfrac{2}{1+e^{-x}} - 1 = \dfrac{1-e^{-x}}{1+e^{-x}}$
Graph		
Range	$0 < f(x) < 1$	$-1 < g(x) < 1$
Relation between f(x) and g(x)	$g(x) = 2f(x) - 1$	
Value of x	$x = \ln\left[\dfrac{f(x)}{1-f(x)}\right]$	$x = \ln\left[\dfrac{1+g(x)}{1-g(x)}\right]$
Derivative	$\dfrac{df(x)}{dx} = e^{-x}/(1+e^{-x})^2$ $= f(x)\left[1-f(x)\right]$	$\dfrac{dg(x)}{dx} = 2e^{-x}/(1+e^{-x})^2$ $= 0.5\left[1-g^2(x)\right]$
Graph of deravitive		

Range of Values of derivative	$0 < \frac{df(x)}{dx} < 0.25$	$0 < \frac{dg(x)}{dx} \leq 0.5$
Relation between $\frac{df(x)}{x}$ and $g(x)/dx$	$\frac{dg(x)}{dX} = 2 \frac{df(x)}{dx}$	

generalization $\Rightarrow f(x) = \frac{1}{1+e^{-\alpha x}}$; $\alpha \equiv$ positive Parameter

* α adds another degree of freedom alongside the weights (a new parameter)

sigmoidal functions of the forms

① $f(x) = \frac{1}{1+e^{-\alpha x}}$

② $g(x) = \frac{2}{1+e^{-\alpha x}} - 1 = \frac{1-e^{-\alpha x}}{1+e^{-\alpha x}}$

③ $h(x) = \tanh(\alpha x)$ where $\alpha \equiv$ Positive Parameter

* $f(x) = \frac{1}{1+e^{-\alpha x}}$; verify that

$$x = \frac{1}{\alpha} \ln \left[\frac{f(x)}{1-f(x)} \right]$$

نفس الصورة السابقة و $\alpha = 1$ بس

- Proof

$$f(x) + f(x)\, e^{-\alpha x} = 1$$

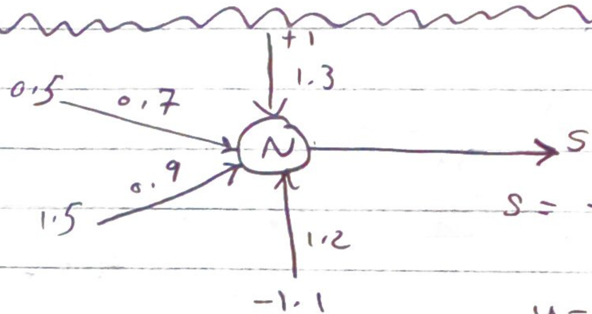
$$e^{-\alpha x} = \frac{1-f(x)}{f(x)}$$

$$-\alpha x = \ln\left[\frac{1-f(x)}{f(x)}\right]$$

$$\alpha x = \ln\left[\frac{f(x)}{1-f(x)}\right]$$

$$x = \frac{1}{\alpha}\ln\left[\frac{f(x)}{1-f(x)}\right]$$

AND $$\alpha = \frac{1}{x}\ln\left[\frac{f(x)}{1-f(x)}\right]$$



$$S = \frac{1}{1+e^{-0.6y}}$$

$y \equiv$ activation

* Activation $y = (0.5)(0.7) + (0.9)(1.5) + (1.2)(-1.1) + (1.3)$
$= 1.68$

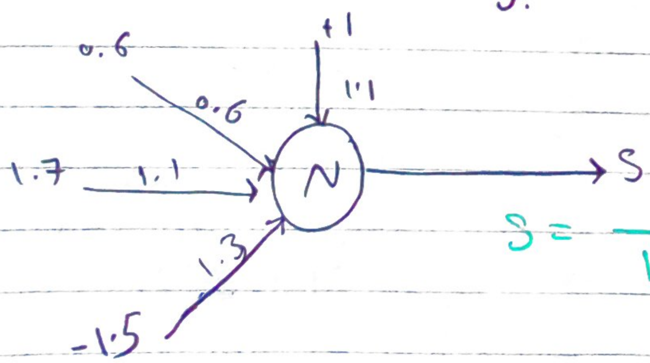
$$S = \frac{1}{1+e^{-0.6(1.68)}} = 0.733$$

$w = 0.6, 1.1, 1.3$; 0.6 ; 1.7 ; -1.5 مدخلات Neuron ليتقبل

$y \equiv$ is activation fn

$\alpha \equiv$ positive paramater

أوجد قيمة α لتصبح قيمة الخرج 0.66 ، اعتبر Bias=1.1



$$S = \frac{1}{1+e^{-\alpha y}} = 0.66$$

Activation

$$y = (0.6)(0.6) + (1.7)(1.1) + (1.5)(1.3) + 1.1$$

$$= 1.38$$

$$\alpha = \frac{1}{y} \ln\left[\frac{f(y)}{1 - f(y)}\right] = \frac{1}{y} \ln\left[\frac{s}{1-s}\right]$$

$$= \frac{1}{1.38} \ln\left[\frac{0.66}{1 - 0.66}\right] = 0.481$$

* Differentiation

$$f(x) = \frac{1}{1 + e^{-\alpha x}}$$

verify that

نفس شكل الجدول ب $\alpha = 1$

$$\frac{d\,f(x)}{dx} = \frac{\alpha e^{-\alpha x}}{(1 + e^{-\alpha x})^2}$$

$$= \alpha\, f(x)\,[1 - f(x)]$$

Proof

$$f(x) = \frac{1}{1 + e^{-\alpha x}}$$

$$\frac{d\,f(x)}{dx} = \frac{\alpha e^{-\alpha x}}{(1 + e^{-\alpha x})^2} \quad (\text{first form})$$

$$= \frac{\alpha}{1 + e^{-\alpha x}} \cdot \frac{e^{-\alpha x}}{1 + e^{-\alpha x}}$$

$$= \frac{\alpha}{1 + e^{-\alpha x}} \cdot \frac{1 + e^{-\alpha x} - 1}{1 + e^{-\alpha x}}$$

$$= \frac{\alpha}{1 + e^{-\alpha x}} \left[1 - \frac{1}{1 + e^{-\alpha x}}\right]$$

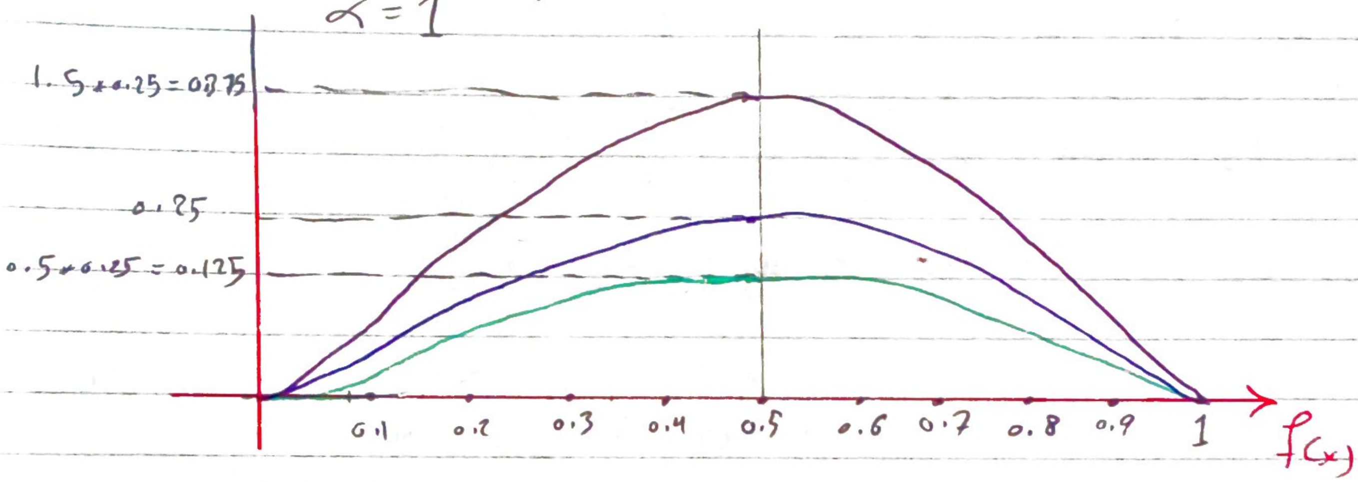
$$= \alpha\, f(x)\,[1 - f(x)]$$

ارسم شكل التفاضل $f(x)$ vs $\frac{df(x)}{dx}$ عند $\alpha = 0.5, 1, 1.5$

أثبت أن القيمة العظمى للتفاضل $\alpha\, 0.25$ وتحدث عندما $f(x) = 1/2$

$$\frac{df(x)}{dx} = \alpha\, f(x)\left[1 - f(x)\right]$$

f(x)	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
df(x)/dx	0	0.09				0.25				0.09	0



* يتضح من هذا الشكل أن القيمة العظمى للمشتقة $\frac{df(x)}{dx}$ هي $\alpha\, 0.25$
وتحدث عندما $f(x) = 0.5$

بغض النظر عن قيمة α فإن $f(x) = 0$ تعطي تفاضل بقيمة 0
" " " " " " $f(x) = 1$ " " " 0

يمكن إثبات هذه النتيجة رياضيا كما يلي

Mathematically,

$$\frac{df(x)}{dx} = \alpha\, f(x)\left[1 - f(x)\right]$$
$$= \alpha\, f(x) - \alpha\, f^2(x)$$

$$\frac{d\left[\frac{df(x)}{dx}\right]}{d f(x)} = \alpha - 2\alpha\, f(x) = 0 \Rightarrow f(x) = 0.5$$

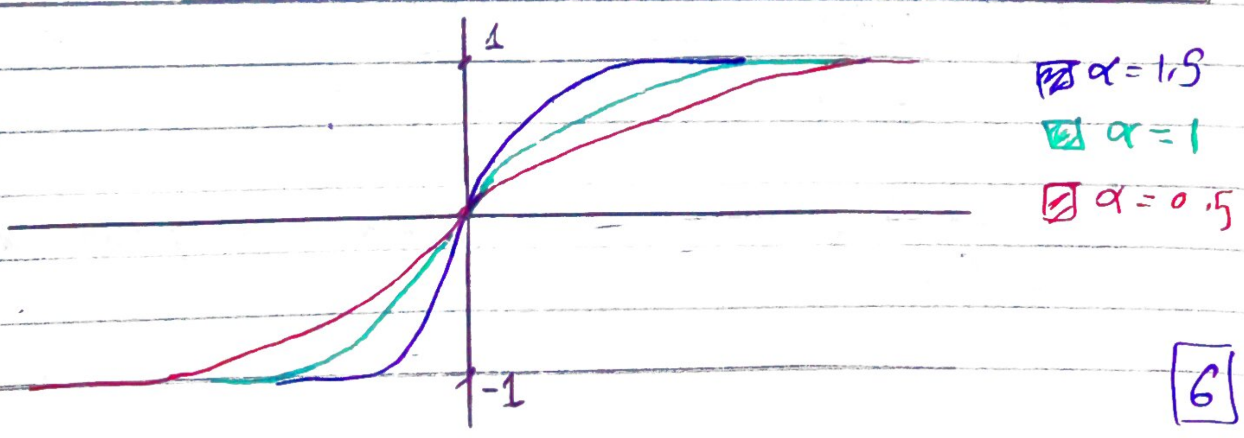
$$\left.\frac{d f(x)}{dx}\right|_{max} = \alpha (0.5)(1-0.5) = 0.25\,\alpha$$

فيك غلطة، انك تحكم بالنتيجة Max أو min. فللتأكد هاتت التفاضل الثاني، لو كان موجب اذا هي min وعكس ذلك تكون min

Generalization $\rightarrow g(x) = \dfrac{2}{1-e^{-\alpha x}} - 1 = \dfrac{1-e^{-\alpha x}}{1+e^{-\alpha x}}$

- Draw on the same coordinates axes, the graphs of $g(x)$ for $\alpha = 0.5, 1$, and 2
- Comment on these graphs
- verify that $x = \dfrac{1}{\alpha} \ln\left[\dfrac{1+g(x)}{1-g(x)}\right]$
- verify that $\dfrac{dg(x)}{dx} = 0.5\,\alpha\,[1-g^2(x)]$

x	$g(x), \alpha=0.5$	$g(x), \alpha=1$	$g(x), \alpha=2$
-5	-0.848	-0.987	-0.999
-4	-0.762	-0.964	-0.998
-3			
-2			
-1			
0	0	0	0
1			
2			
3			
4	0.762	0.964	0.998
5	0.848	0.987	0.999



* مع زيادة قيم α فإن الشكل البياني يقترب أكثر وأكثر من المحور الرأسي لعندما تؤول α ← ∞ فإن الدالة تصبح Bipolar threshold

$g(x) = \dfrac{1-e^{-\alpha x}}{1+e^{-\alpha x}}$

$$g(x) + e^{-\alpha x} g(x) = 1 - e^{-\alpha x}$$

$$e^{-\alpha x} = \frac{1-g(x)}{1+g(x)} \Rightarrow -\alpha x = \ln\left[\frac{1-g(x)}{1+g(x)}\right]$$

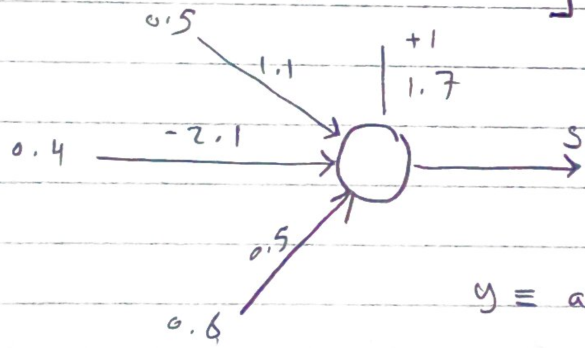
$$\Rightarrow x = \frac{1}{\alpha} \ln\left[\frac{1+g(x)}{1+g(x)}\right]$$

$\dfrac{dg(x)}{dx} = \dfrac{(1+e^{-\alpha x})(\alpha e^{-\alpha x}) - (1-e^{-\alpha x})(-e^{-\alpha x})}{(1+e^{-\alpha x})^2}$

$$= 0.5\,\alpha \left[1 - \left(\frac{1-e^{-\alpha x}}{1+e^{-\alpha x}}\right)^2\right]$$

$$0.5\,\alpha \left[1 - g^2(x)\right]$$

#



$$s = \frac{2}{1+e^{-\alpha y}} - 1$$

y = activation

Find the value of the parameter α such that $s = 0.75$

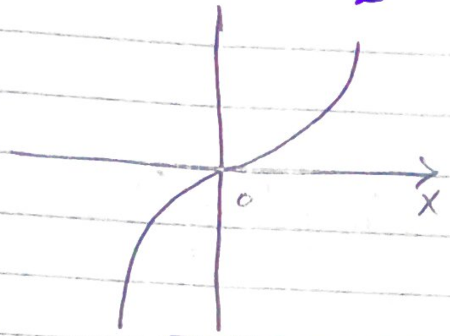
Activation $y = (0.5)(1.1) + (0.4)(-2.1) + (0.6)(0.5) + 1.7 = 1.71$

$$y = \frac{1}{\alpha} \ln \frac{1+s}{1-s} \Rightarrow \alpha = \frac{1}{y} \ln \frac{1+s}{1-s}$$

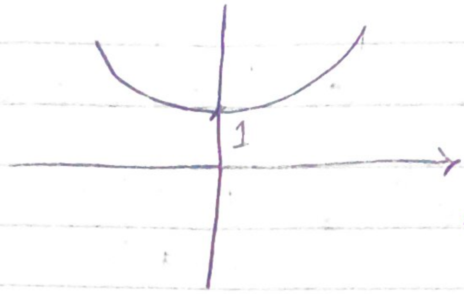
$$\alpha = \frac{1}{1.71} \ln \frac{1+0.75}{1-0.75} = 1.138$$

* Hyperbolic functions

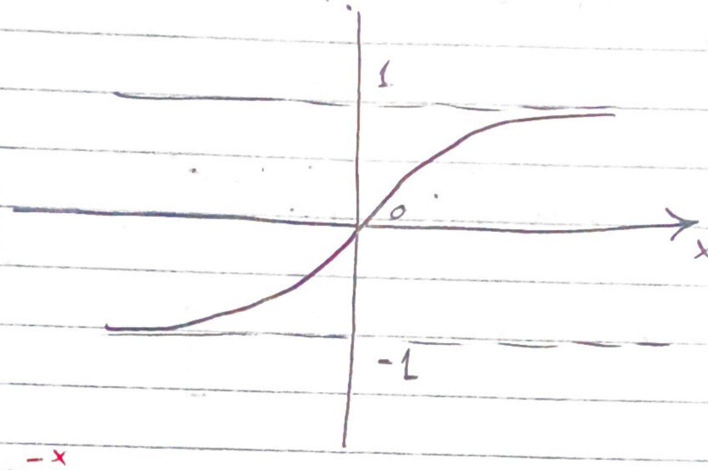
$$\sinh x = \frac{e^{x} - e^{-x}}{2}$$



$$\cosh x = \frac{e^{x} + e^{-x}}{2}$$



$$\tanh x = \frac{\sinh x}{\cosh x} \Rightarrow \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$$



$$\tanh x = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$$

$$= \frac{1 - e^{-2x}}{1 + e^{-2x}}$$

$$= \frac{2}{1 + e^{-2x}} - 1 = \text{bipolar Sigmoid at } \alpha = 2$$

$\tanh x$ = bipolar sigmoid at $\alpha = 2$

Generally,

$$\frac{2}{1 + e^{-\alpha x}} - 1 = \frac{1 - e^{-\alpha x}}{1 + e^{-\alpha x}} \Longrightarrow$$

$$= \frac{e^{\frac{\alpha x}{2}} - e^{-\frac{\alpha x}{2}}}{e^{\frac{\alpha x}{2}} + e^{-\frac{\alpha x}{2}}} = \tanh\left(\frac{\alpha x}{2}\right)$$

* وضع $\alpha = 2$ نحصل على النتيجة السابقة

$$\text{Bipolar Sigmoid} = \frac{2}{1 + e^{-\alpha x}} - 1 = \tanh\left(\frac{\alpha x}{2}\right)$$

نستنتج أنه يتم استخدام tanh لـ Bipolar f_n